Using Aesthetic Measures to evolve Art

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***Abstract*— In this paper we investigate and compare three aesthetic measures within the context of evolutionary art. We evolve visual art with an unsupervised evolutionary art system using genetic programming and an aesthetic measure as the fitness function. We perform multiple experiments with different aesthetic measures and examine their influence on the evolved images. Additionally, we perform a cross-evaluation by calculating the aesthetic value of images evolved by measure i according to measure j. This way we investigate the flexiblity of each aesthetic measure (i.e., whether the aesthetic measure appreciates different types of images). Last, we perform an image analysis using a fixed set of image statistics functions. The results show that aesthetic measures have a rather clear 'style' and that these styles can be very different. Furthermore we find that some aesthetic measures show little flexibility and appreciate only a limited set of images. The images in this paper might only be in color in the electronic version.**

I. Introduction

The goal of the research field of Computational Aesthetics is to investigate "computational methods that can make applicable aesthetic decisions in a similar fashion as humans can" [8]. Aesthetic measures are functions that compute the aesthetic value of an object. Birkhoff was the first to publish on the subject of aesthetic measures (see [3]), and his work has been influential in the field. Birkhoffs notion of aesthetics was based on the relation between Order and Complexity, expressed as $M = \frac{O}{C}$, where O stands for order and C for Complexity. Birkhoffs measure is now widely regarded a being mostly a measure of orderliness. Since Birkhoff, several researchers have investigated aesthetic measures from several points of view. [7] and [8] give good overviews of the field.

A. Research question

In this paper we investigate and compare three aesthetic measures. This paper can be seen as a sequel to [6], in which we investigated three other aesthetic measures and a combination measure in our evolutionary art system. Each aesthetic measure is used in an evolutionary art system as a fitness function (all evolutionary parameters are kept equal for all aesthetic measures). We evolve small Lisp like expressions that generate images, and compare the difference between the images created by the three aesthetic measures. Next, we investigate how the produced images using aesthetic measure M_i are judged by the other aesthetic measures. Hereby we obtain an indication of the neutrality of the measure (aesthetic measures that only give high score to images produced using the aesthetic measure itself have a limited 'scope'). Last, we calculate a number of statistics of image properties (like average brightness, saturation etc.) of the produced images.

The rest of the paper is structured as follows. First we discuss evolutionary art and the use of aesthetic measures within the context of evolutionary art (section II). Section III discusses our software environment Arabitat. Next, we describe the experiments and their results in section IV. Sections V and VI contain conclusions and directions for future work.

II. Evolutionary art

Evolutionary art is a research field where methods from Evolutionary Computation are used to create works of art (good overviews of the field are [17] and [2]). Some evolutionary art systems use supervised fitness assignment (e.g. [20], [18]), and in recent years there has been increased activity in investigating unsupervised fitness assignment (e.g. [1], [12], [19]). The field of Computational Aesthetics investigates how computational methods can be used to assign aesthetic judgement to objects (see [8] and [7]). Functions that assign an aesthetic value to an object are typically called aesthetic measures. In this paper we investigate three aesthetic measures, and compare their output. To our knowledge, this is one of the few attempts to systematically investigate the workings of multiple aesthetic measures in the context of evolutionary art.

A. Aesthetic measures

First we shortly describe the aesthetic measures that were used in our experiments. The aesthetic measures are (in alphabetical order) Benford Law [9], Global Contrast Factor [13] and Information Theory [16]. In the next subsections we will give a brief description of each aesthetic measures; more details can be found in the original papers.

1) Benford Law: We implemented a simple aesthetic measure based on Benford Law (see [9], [5]); Benford Law (or first-digit law) states that list of numbers obtained from real life (i.e. not created by man) are distributed in a specific, non-uniform way. The leading digit occurs one third of the time, the second digit occurs 17.6%, etc. (see Figure 1).

We use Benford law to measure the distribution of luminance (brightness) of pixels. For an image we calculate the brightness histogram using 9 bins. Next we calculate the difference between the actual histogram and the Benford histogram;

$$M_{benford} = \frac{d_{max} - d_{total}}{d_{max}} \tag{1}$$

where d_{total} is

$$d_{total} = \sum_{i=1}^{9} \left(\frac{H_{image}(i)}{N} - H_{benford}(i)\right)^p \tag{2}$$

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d	p
1	30.1%
2	17.6%
3	12.5%
4	9.7%
5	7.9%
6	6.7%
7	5.8%
8	5.1%
9	4.6%

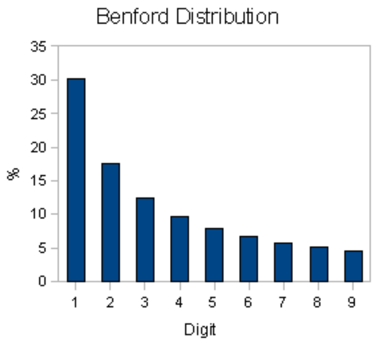

Fig. 1. The Benford distribution

where $H_{image}(i)$ is the number of entries in the luminance histogram bin number i and N is the total number of pixels in the image. $H_{benford}(i)$ is the value from the Benford distribution (see Figure 1). The maximal difference d_{max} for $p = 3$ is $(1 - 0.301)^3 + (0.176)^3 \ldots + (0.046)^3 = 0.3511$. Lower values for p (we experimented with $p = 3$, $p = 2$ and $p = 1$) result in a higher penalty for differences in brightness distribution. For our experiments we used $p = 1$.

2) Global Contrast Factor: The Global Contrast Factor is an aesthetic measure described in [13]. Basically, the global contrast factor computes contrast (difference in luminance or brightness) at various resolutions. Images that have little or few differences in luminance have low contrast and are considered ‘boring’, and thus have a low aesthetic value. Contrast is computed by calculating the (average) difference in luminance between two neighbouring superpixels. Superpixels are rectangular blocks in the image. The contrast is calculated for several resolutions (2, 4, 8, 16, 25, 50, 100 and 200) and the average contrast is summed as

$$M_{gcf}(I) = \sum_{k=1}^{9} w_k \cdot contrast(n, p_k, r_k) \tag{3}$$

where r_k refers to the resolution of the superpixel, w_k refers to the weight of the contrast of the superpixel (the weight of the contrast differs per resolution) and p_k is a power factor. Both w and p were optimised using several experiments in [13]. In our implementation we used all the settings from [13], and we refer to that paper for more details.

3) Information Theory: There have been several efforts to use information theory to calculate the aesthetic value of an object. [8] and [7] describe a number of methods by Bense and Moles, and [16] describe a family of closely related aesthetic measures funded on Shannon entropy and Kolmogorov complexity. Our information theory aesthetic measure is an implementation of [16], whereby we have implemented the variant using Kolmogorov complexity using RGB entropy;

$$M_{it}(I) = \frac{NH_{max} - K}{NH_{max}} \tag{4}$$

where N is the image size (the number of pixels) and H_{max} is a constant colour length code which is 24 in our case (since we use 24 bit colour; 8 bits for each R,G,B channel). K_{max} stands for Kolmogorov complexity of the image. Since Kolmogorov complexity can only be estimated, we (like [16]) use JPEG compression. In our implementation, we used a JPEG quality setting of 75%. For more details and for other variants of this aesthetic measure we refer to [16].

III. ARABITAT: THE ART HABITAT

Arabitat (Art Habitat) is our software environment in which we investigate evolutionary art. It uses genetic programming with Lisp expressions and supports both supervised and unsupervised evaluation. In this paper we only discuss unsupervised fitness evaluation using aesthetic measures. Currently we have implemented seven aesthetic fitness functions and a weighted sum combination measure (see [6] for experiments with three other aesthetic measures), and intend to implement more in the near future. In our system, a genotype consists of 1) a Lisp-style expression that returns a value of type ‘double’, and 2) a color lookup table. Lisp-like expressions are common within genetic programming (see [11]). Our genetic programming is type-safe and returns only results of type ‘double’.
The computation of a phenotype from the genotype is done as follows; for a target phenotype image with a resolution *(width, height)* we calculate the function value from the lisp expression (the genotype) for each *(x,y)* coordinate of the image. The Lisp expression is subject to crossover and mutation; we use standard subtree crossover and standard subtree mutation (see [11]). The resulting matrix of floating points is mapped onto an indexed colour table, and this results in a matrix of integers, where each integer refers to a colour index of the corresponding colour scheme. This way the colouring is independent of the double values (other approaches like [20] have functions in the function set that directly address colouring). The colour scheme is also part of the genotype, and is also subject to mutation and crossover. A mutation in the colour scheme could result in an entirely different coloured image, even if the expression remain unaltered. The resulting image is passed to the fitness function (one of the aesthetic measures) for evaluation. See Figure 2 for a schematic overview (see http://www.few.vu.nl/˜eelco/ for more examples in colour).

A. Function set

Table I contains an overview of all the terminals and functions that we used in our experiments. The terminals x and y are variables that refer to the (x, y) coordinate of a pixel. ‘Width’ and ‘height’ are variables that refer to the width and height of the image. The use of width and height is useful because we usually perform evolutionary computation using images with low resolution (say 300x300) and want to display the end result on a higher resolution. The ‘Basic math’, ‘Other math’, ‘Relational’, ‘Conditional’ and ‘Bitwise’ functions mostly speak for themselves and are described in [20] and [18]. Most ‘Noise’ functions are from [20] except for ‘moire’, which was taken from [15];

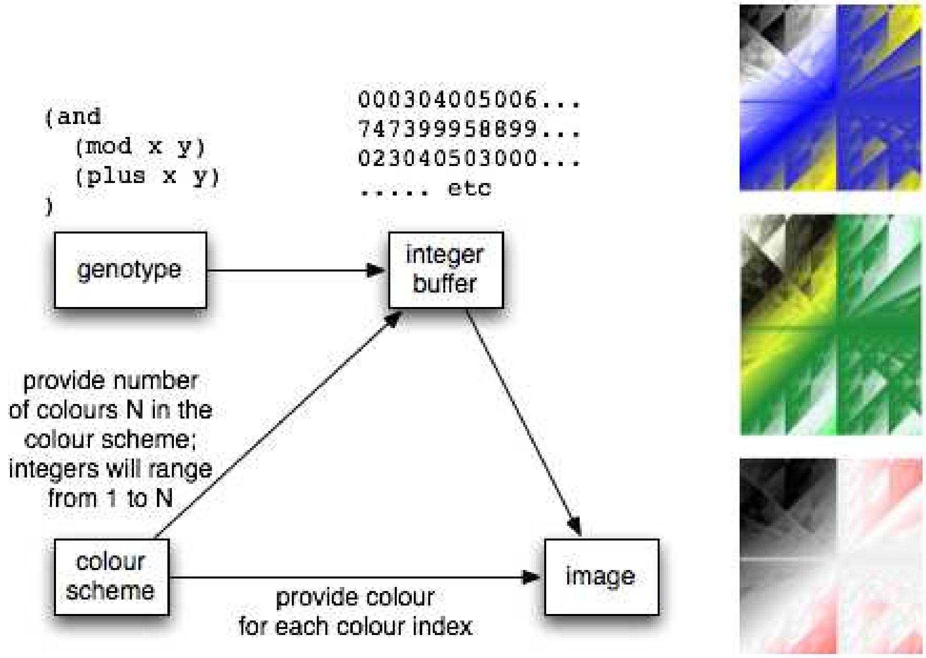

Fig. 2. A schematic overview of the expression of the genotype into the phenotype (image) for LISP expression ((and (mod x y) (plus x y))); the three images on the right are three renderings of the same expression, using three different colour schemes.

Terminals	x,y, ephem_double, ephem_int, width, height, golden_ratio, pi
Basic math	plus/2, minus/2, multiply/2, div/2, mod/2, minimum/2, maximum/2, abs/1, average/2, log/1
Other math	sin/1, cos/1, tan/2, sinh/1, cosh/1, hypot/2
Relational	lessthan/2, greaterthan/2, equals/2
Conditional	ifthenelse/3
Bitwise	and/2, or/2, xor/2
Noise	marble/2, turbulence/2, plasma/2, moire/2
Fractal	mandelbrot/2, julia/2, mandeltweak/2
Chaos	complexiteratormap/2, chaoticdust/2, spiralform/2, chaosbits/2

TABLE I

FUNCTION AND TERMINAL SET OF OUR EVOLUTIONARY ART SYSTEM

it generates a so-called moire pattern; a semi-random semi-repetitive noisy pattern. The 'mandelbrot' and 'julia' function refer to implementations of the well-known Mandelbrot set and Julia set. The 'mandeltweak' is a specialized or 'tweaked' variant of the Mandelbrot set and was taken from [21]. The paper [21] describes a wide range of parameters that can be tweaked to create chaos-like Mandelbrot-like figures. For performance reasons we have manually tweaked one Mandelbrot-like figure, and re-use the settings of that figure. All the 'Chaos' functions come from [15].

IV. EXPERIMENTS

In order to investigate and compare the three different aesthetic measure we conducted a number of experiments. We performed 10 runs for each aesthetic measure and collected the images of the 5 fittest individuals of each run. Next, we calculated the aesthetic measure of those 5 individuals by the other aesthetic measures. From the 50 images of each experiment (10 runs, 5 fittest individuals) we handpicked 9 images that were typical for that image set. Besides the aesthetic measure, all evolutionary parameters were the same for each run. We did many preliminary experiments and found that populations of around 200 usually tended to converge to one or two dominant individuals and their similar offspring. Since the goal of this paper is to compare the output of evolutionary art using different aesthetic measures, we decided to perform evolutionary search for 10 generations with a population of 200. For the genetic operators we used subtree mutation (rate 0.15), subtree crossover (rate 0.85), we initialized the population using the well-known ramped half-and-half initalization method (see [11]), and used tournament selection (tournament size 3) for both parent selection and survivor selection. For survivor selection we use elitist selection (best 1). Initial experiments have shown that a lot of time is spent on genotypes that produce extremely simple images (mostly images with two or more single-color bands). In order to avoid unnecessary search, we introduced a minimal complexity threshold of 3%; an image that can be compressed using PNG to 3% or less of its original size is discarded; its fitness is set to 0 and the genotype will most likely be replaced by a fitter individual in the next generation. This simple threshold rule greatly increases the quality of the output images, although it does introduce a bias; Mondriaan type images, or images like the works of Malevich' 'Black square' (1915) and or works from the art movement known as 'Suprematism' are probably outside the scope of our system.

A. Results: manual selection

1) Benford Law: Figure 4 show the average fitness using the Benford Law aesthetic measure as the fitness function.

We hand-picked 9 images from the resulting 50 images and they are presented in Figure 3. The images seem to have a preference for dark colours, where brown seems to be popular. The image texture of the Benford Law images are varied.

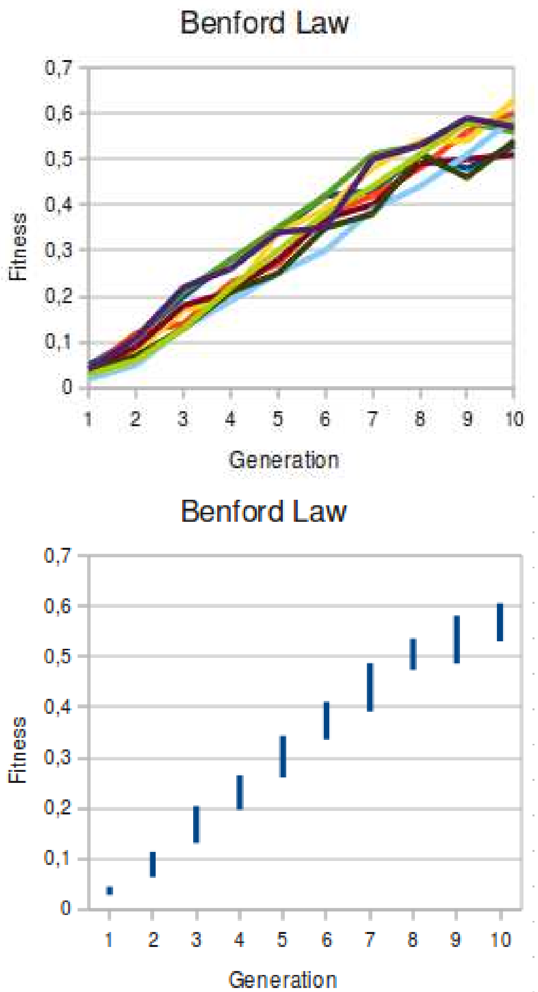

Fig. 4. Fitness progression of 10 different runs using the Benford Law aesthetic measure (10 generations); the upper graph shows the 10 individual runs; the lower graph shows the average fitness plus/ minus the standard deviation for the 10 experiments. Fitness scale is between 0 and 0.7

2) Global Contrast Factor: The Global Contrast Factor calculates and values contrast on various resolutions of an image, and this results (as expected) in images with a lot of contrast.

Most images have little color variation (Figure 5, and contain high contrast colours (shades of black, shades of white). Since contrast is calculated at various resolutions, the spread of contrast across different resolutions is rewarded, and this results in lively images.

3) Information Theory: The information theory aesthetic measure [16] optimizes images that have a low JPEG compression ratio. Images evolved using this measure will therefore have the tendency to be relatively simple. Since image size is the only relevant driving factor, the resulting

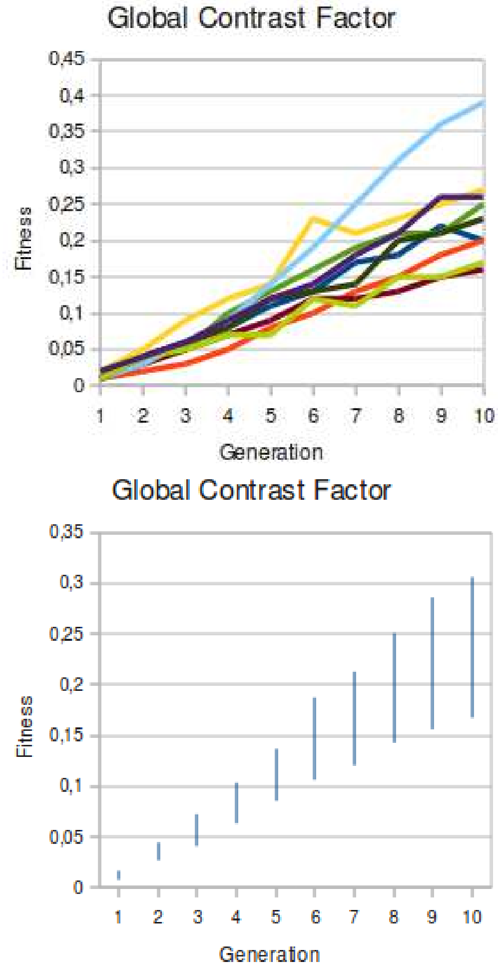

Fig. 6. Fitness progression of 10 different runs using the Global Contrast Factor (10 generations); the upper graph shows the 10 individual runs; the lower graph shows the average fitness plus/ minus the standard deviation for the 10 experiments. Fitness scale is between 0 and 0.45

images are very diverse in texture and colour. The original resulting images have a very low contrast value; we therefore show the original images in Figure 7 and we show the same images with the contrast increased (Figure 8).

B. Cross-evaluation

After we had done the experiments with the three aesthetic measures, we wanted to know how the aesthetic measures would evaluate ‘each others’ work. The evaluation of the work of measure M_i of images produced using aesthetic measure M_j might give us an indication of the ‘scope’ of the aesthetic measure. If an aesthetic measure only appreciates images that were generated using its own measure, then we could assume that its scope were fairly limited. On the other hand, if a measure also appreciates images that were created using another aesthetic measure, we could conclude that it is applicable to a broader scope of images. In Table II we have gathered (for each of the three aestetic measures) the average fitness and standard deviation of the fifty fittest individuals that were collected for each experiment.

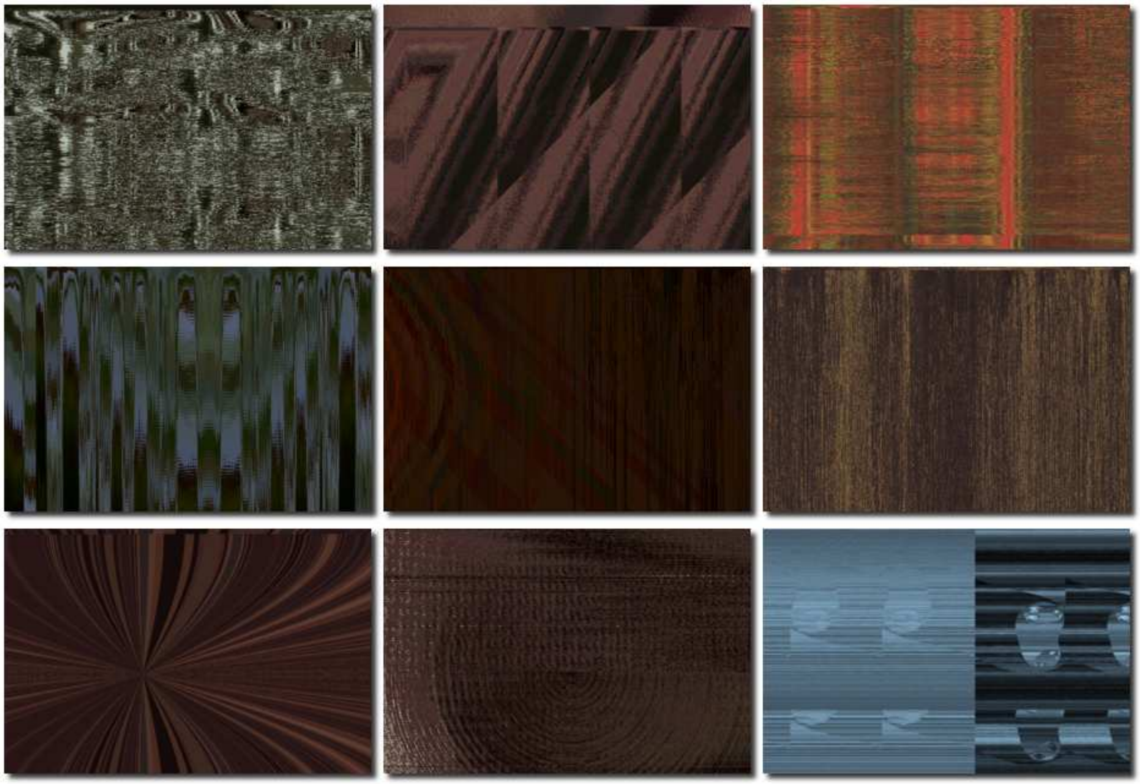

Fig. 3. Summary of images evolved using the aesthetic measure of Benford Law

Fig. 5. Summary of images evolved using the aesthetic measure of the Global Contrast Factor

Fig. 7. Summary of the images evolved using the Information theory aesthetic measure

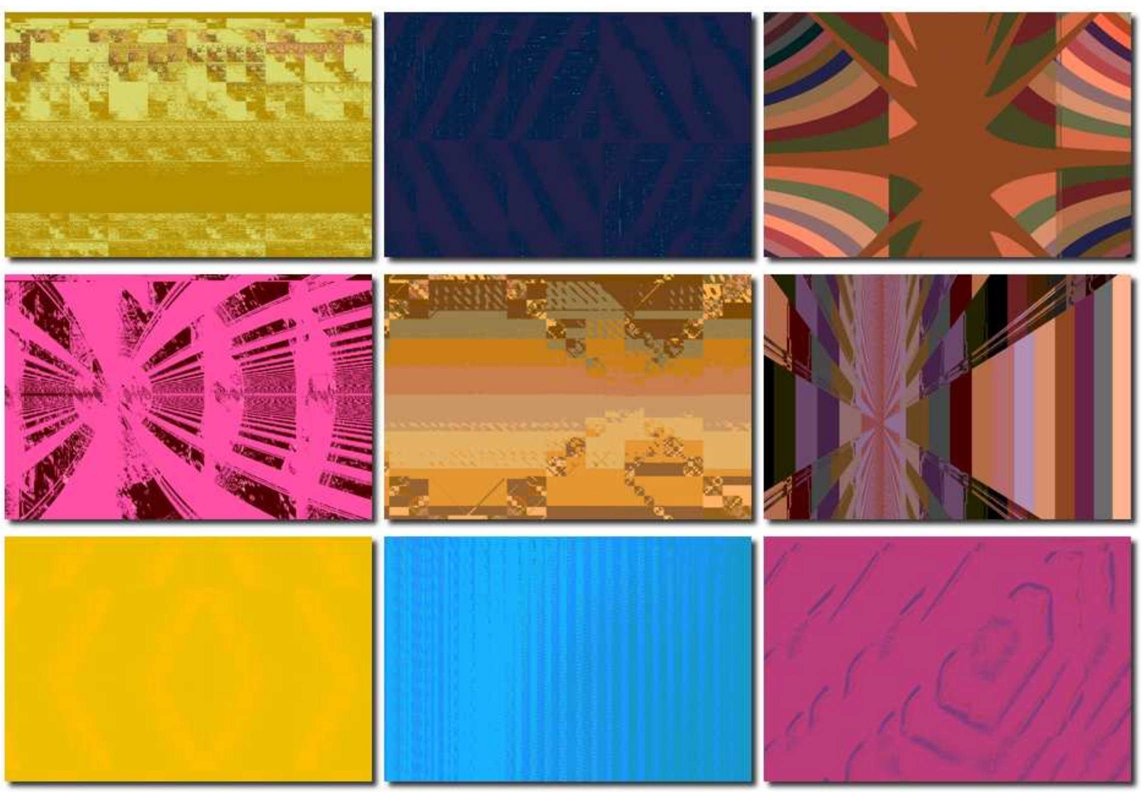

Fig. 8. Summary of images evolved using the Information theory aesthetic measure (Figure 7) with contrast enhanced

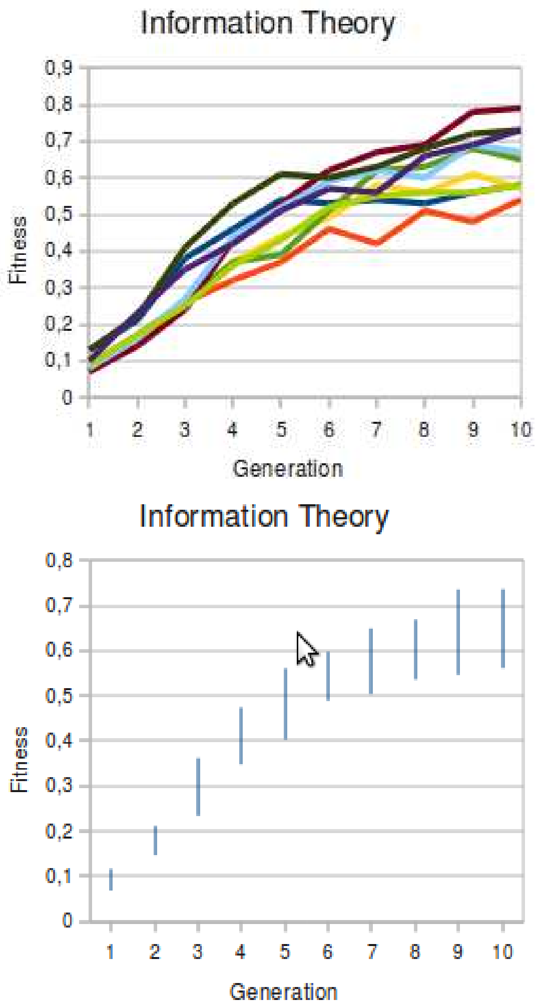

Fig. 9. Fitness progression of 10 different runs using the Information Theoy aesthetic measure (10 generations); the upper graph shows the 10 individual runs; the lower graph shows the average fitness plus/ minus the standard deviation for the 10 experiments. Fitness scale is between 0 and 0.9

		Evaluated by		
		Benford Law	GCF	Information Theory
	Benford Law	.431 (.364)	.064 (.081)	.408 (.364)
Produced	GCF	.235 (.227)	.405 (.371)	.353 (.294)
By	Inf. Theory	.023 (.063)	.001 (.001)	.696 (.516)

TABLE II

THE CROSS EVALUATION OF THE AESTHETIC VALUE OF EACH OTHERS IMAGES. WE PRESENT THE MEAN AESTHETIC VALUE AND THE STANDARD DEVIATION IN PARENTHESES

	Aesthetic Measure		
	Benford Law	GCF	Information Theory
Mean Hue	93.9 (89.5)	49.3 (60.5)	163 (56.7)
Min. Hue	47.5 (70.5)	4.3 (11)	136.7 (55.2)
Max. Hue	150.3 (92.1)	106.9 (85.7)	187.8 (58.3)
Mean Saturation	122.4 (58.4)	63.6 (79.2)	48.8 (50.1)
Min. Saturation	69.6 (77.9)	16.3 (61.6)	24.7 (33.8)
Max. Saturation	192.4 (75.1)	120.7 (111.1)	93.8 (90.4)
Mean Brightness	95.2 (36.5)	92.7 (53.8)	239.4 (27.2)
Min. Brightness	23.9 (18.4)	18 (46.6)	229.6 (36.9)
Max. Brightness	182.3 (77.7)	233.1 (49.4)	254.6 (1.7)
Mean Red	84.8 (40.9)	89.1 (53.2)	209.5 (55.6)
Min. Red	20.2 (20.7)	12.9 (46)	178.9 (82.9)
Max. Red	162.8 (83.5)	231.4 (50.1)	228.7 (42.8)
Mean Green	61.2 (36)	85.7 (55.4)	218.7 (39.8)
Min. Green	15.1 (15.7)	10.5 (45.9)	208.8 (52.4)
Max. Green	125.7 (78.4)	228.9 (60.7)	234.8 (24.3)
Mean Blue	71.7 (40)	84.6 (53.6)	222.9 (39.2)
Min. Blue	12.7 (13.2)	14.9 (46.8)	199.9 (75.3)
Max. Blue	147.4 (80.3)	221.6 (63.7)	240.3 (32.1)

TABLE III

IMAGE STATISTICS PER AESTHETIC MEASURE; WE PRESENT THE MEAN VALUE FOR EACH STATISTICS AND THE STANDARD DEVIATION BETWEEN PARENTHESES.

The producing aesthetic measure is presented horizontally and the evaluation by all aesthetic measures is presented in the columns. From this table we can conclude a number of findings. First, it is apparant that all aesthetic measures like 'their own' images best. This is not surprising, but it also show that none of the three aesthetic measures is very open to to other styles of images. Next, we see that the images produced using the Information Theory aesthetic measures are not appreciated by the other two aesthetic measures. The Global Contrast Factor measure gave an average fitness value of 0.001 and the Benford aesthetic measure gave an average fitness value of 0.023. The images produced by the Information Theory measure have little contrast and have a high mean brightness per pixel (also see the next section). The first property (low contrast) causes a low score from the GCF aesthetic measure. The second property (high mean brightness per pixel) indicates that brightness values are rather uniform, resulting in a large difference with the Benford distribution. We can also conclude that the Global Contrast Factor aesthetic measure has a limited 'scope'; it gives high average scores to its own images (0.405) but gives low scores to images produced by the other measures.

C. Image statistics

In previous sections we gave a qualitative description of the images that were a result of evolution using one the three aesthetic measures as the fitness functions. Although qualitative assessment is valuable, it can also be very useful to give a quantitative assessment. Here we give a small statistical overview of a number of image properties, and group them by the aesthetic measure that 'produced' it. Of the 50 images of each aesthetic measure we calculated the mean, maximum, and minimum for the image properties hue, saturation, brightness, red, green. We calculated these properties for each image, and then calculated the mean value, and the standard deviation for each. All image properties and their statistics are described in Table IV-C.

From the image statistics in Table IV-C we can conclude the following. First of all; the Global Contrast Factor aesthetic measure ensures that its produced images have brightness values that maximize the contrast; the average image produced using the Global Contrast Factor has an average brightness between 18 and 233.1, whereas the average image produced using the Information Theory aesthetic measure has an average brightness between 229.6 and 254.6 (the absolute maximum is 255). From the latter observation, we can conclude that the average image produced by the Information Theory aesthetic measure is 1) very bright and 2) has little contrast. Both observations were also done in Section IV after inspection of the images in Figure 7.

V. Conclusions

In this paper we have investigated and compared three aesthetic measures in an evolutionary art system. After our experiments we can conclude that the use of different aesthetic measures clearly results in different 'styles' of evolutionary art. Since all evolutionary parameters were kept equal in all experiments, we can conclude that all differences in artistic style are directly related to the aesthetic measures. Next, we can conclude that there are also differences in variety of the output of the three aesthetic measures. The differences in contrast between the images produced by the Global Contrast Factor measure and the Information Theory measure is striking. These differences are also apparant from the statistic analysis of the image properties. Images produced using the Information Theory aesthetic measure could probably benefit from a simple contrast enhancement; without this, the images tend to be a bit dull. Last, we can conclude that of all aesthetic measures in our experiments, the Global Contrast Factor has the narrowest 'scope'; it only gives high scores to its own images, and very low scores to imges produced by other aesthetic measures. Nevertheless, we liked the images produced by the Global Contrast Factor, and we think that the GCF aesthetic measure can be very useful in a multi objective optimization setup with multiple aesthetic measures.

VI. Future work

In this paper we chose three aesthetic measures as input for experiments with evolutionary art. Furthermore, there exist more aesthetic measures in literature. We have already tested an implementation of the Pattern Measure of [10], but the computation of this aesthetic measure is very slow (about 100 times slower than the computation of the aesthetic measures that we used in our experiments). We will investigate whether we can speed up the computation of this aesthetic measure, or we will have to reserve a lot of time. Next, we would like to further explore the combination of multiple aesthetic measures into a combined aesthetic measure using techniques from multi-objective optimization (see [4]). Also, we would like to improve the diversity in our populations in order to avoid premature convergence caused by a small group of fit individuals. There exist several solutions to cope with premature convergence and we intend to investigate them. We would also like to investigate the role of representation in evolutionary art. It would be interesting to choose a representation other than the quintessential Lisp representation, and to compare several runs of different representations using the same aesthetic measure, in order to compare the influence of the choice of the representation.

In this paper we calculated a number of image statistics to summarize image properties per aesthetic measure. We think that the use of image statistics is useful, but we think that there are more possibilities in calculating statistics of image properties.

References

[1] Shumeet Baluja, Dean Pomerleau, and Todd Jochem. Towards automated artificial evolution for computer-generated images. *Connection Science*, 6:325–354, 1994.

[2] P. J. Bentley and D. W. Corne, editors. *Creative Evolutionary Systems*. Morgan Kaufmann, San Mateo, California, 2001.

[3] George D. Birkhoff. *Aesthetic Measure*. Harvard University Press, 1933.

[4] K Deb. *Multi-objective Optimization using Evolutionary Algorithms*. Wiley, Chichester, UK, 2001.

[5] Esteve del Acebo and Mateu Sbert. Benford's law for natural and synthetic images. In Neumann et al. [14], pages 169–176.

[6] E. den Heijer and A.E. Eiben. Comparing aesthetic measures for evolutionary art. In *EvoApplications*, Lecture Notes in Computer Science, Berlin, Heidelberg, 2010. Springer.

[7] Gary Greenfield. On the origins of the term "computational aesthetics". In Neumann et al. [14], pages 9–12.

[8] Florian Hoenig. Defining computational aesthetics. In Neumann et al. [14], pages 13–18.

[9] Jean-Michel Jolion. Images and benford's law. *Journal of Mathematical Imaging and Vision*, 14(1):73–81, 2001.

[10] Allen Klinger and Nikos A. Salingaros. A pattern measure. *Environment and Planning B: Planning and Design*, 27:537–547, 2000.

[11] John R. Koza. *Genetic Programming: On the Programming of Computers by Means of Natural Selection*. MIT Press, Cambridge, MA, 1992.

[12] Penousal Machado and Amílcar Cardoso. All the truth about nevar. *Applied Intelligence*, 16(2):101–118, 2002.

[13] Kresimir Matkovic, László Neumann, Attila Neumann, Thomas Psik, and Werner Purgathofer. Global contrast factor-a new approach to image contrast. In Neumann et al. [14], pages 159–168.

[14] László Neumann, Mateu Sbert, Bruce Gooch, and Werner Purgathofer, editors. *Computational Aesthetics 2005: Eurographics Workshop on Computational Aesthetics in Graphics, Visualization and Imaging 2005, Girona, Spain, May 18-20, 2005*. Eurographics Association, 2005.

[15] Clifford A. Pickover. *Computers, Pattern, Chaos and Beauty: Graphics from an Unseen World*. St. Martins Press, New York, 1990.

[16] Jaume Rigau, Miquel Feixas, and Mateu Sbert. Informational aesthetics measures. *IEEE Computer Graphics and Applications*, 28(2):24–34, 2008.

[17] Juan Romero and Penousal Machado, editors. *The Art of Artificial Evolution: A Handbook on Evolutionary Art and Music*. Natural Computing Series. Springer Berlin Heidelberg, November 2007.

[18] Steven Rooke. Eons of genetically evolved algorithmic images. In Bentley and Corne [2], pages 339–365.

[19] B. Ross, W. Ralph, and H. Zong. Evolutionary image synthesis using a model of aesthetics. In *IEEE Congress on Evolutionary Computation (CEC) 2006*, pages 1087–1094, 2006.

[20] Karl Sims. Artificial evolution for computer graphics. In *SIGGRAPH '91: Proceedings of the 18th annual conference on Computer graphics and interactive techniques*, volume 25, pages 319–328. ACM Press, July 1991.

[21] Jeffrey Ventrella. Evolving the mandelbrot set to imitate figurative art. In Philip F. Hingston, Luigi C. Barone, and Michalewicz Zbigniew, editors, *Design by Evolution: Advances in Evolutionary Design*, pages 145–168. Springer Berlin Heidelberg, 2008.